Math 115

Summer 2017

Lecture 15

Ch. 6: Working with Rational Expression

what is a rational expression?

$$\frac{\text{Polynomial}}{\text{Polynomial}}$$

$$\frac{2x-10}{3x-15}, \quad \frac{x^2-4}{x^2+2x-15}, \quad \frac{3x^2+5x-8}{2x^2+3x-5}$$

$$\frac{x^3-125}{x^2-25}$$

How to Simplify rational expression:

1) factor numerator and denominator completely.

2) Cross-out any common factor

Simplify $\dfrac{2x-10}{x^2-25} = \dfrac{2(x-5)}{(x+5)(x-5)} = \boxed{\dfrac{2}{x+5}}$

Simplify $\dfrac{x^2-9}{x^2+8x+15} = \dfrac{(x+3)(x-3)}{(x+5)(x+3)} = \boxed{\dfrac{x-3}{x+5}}$

Simplify: $\dfrac{3x^3-75x}{2x^3-250}$

A^2-B^2

$= \dfrac{3x(x^2-25)}{2(x^3-125)} = \dfrac{3x(x+5)(x-5)}{2(x-5)(x^2+5x+25)}$

$x^3-5^3 = (x-5)(x^2+5x+25)$ → A^3-B^3

$= \boxed{\dfrac{3x(x+5)}{2(x^2+5x+25)}}$

Simplify: $\dfrac{4x-28}{x^4-49x^2}$

$= \dfrac{4(x-7)}{x^2(x^2-49)} = \dfrac{4(x-7)}{x^2(x+7)(x-7)} = \boxed{\dfrac{4}{x^2(x+7)}}$

Simplify: $\dfrac{2x^2 - x - 3}{2x^3 - 3x^2 + 2x - 3}$

$= \dfrac{(2x-3)(x+1)}{x^2(2x-3) + 1(2x-3)}$

$= \dfrac{\cancel{(2x-3)}(x+1)}{\cancel{(2x-3)}(x^2+1)}$

$= \boxed{\dfrac{x+1}{x^2+1}}$

$2x^2 - x - 3$

$P = -6$

$-6 \quad S = -1$

$-3, 2$

$= 2x^2 - 3x + 2x - 3$

$= x(2x-3) + 1(2x-3)$

$= (2x-3)(x+1)$

Simplify: $\dfrac{x+5}{x^2-4x-45} = \dfrac{1(x+5)}{(x-9)(x+5)} = \boxed{\dfrac{1}{x-9}}$

$\dfrac{x^2-4}{x^3+8} = \dfrac{x^2-2^2}{x^3+2^3} = \dfrac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x^2-2x+4)} = \boxed{\dfrac{x-2}{x^2-2x+4}}$

$A^2 - B^2$

$A^3 + B^3$

Simplify

$$\frac{4x^2 - 25}{4x^2 - 20x + 25} = \frac{(2x)^2 - 5^2}{(2x)^2 - 2(2x)(5) + (5)^2}$$

$A^2 - B^2$

$A^2 - 2AB + B^2$

$$= \frac{(2x+5)(2x-5)}{(2x-5)^2}$$

$$= \frac{(2x+5)(2x-5)}{(2x-5)(2x-5)}$$

$$= \boxed{\frac{2x+5}{2x-5}}$$

Excluded Values are those values that make the denominator equal to zero.

$\frac{2x-5}{x-3}$ E.V.: 3

why? $3-3=0$

$\frac{x+7}{(x-5)(x+10)}$

E.V. 5 & -10

$(5-5)(5+10)=0$

$(-10-5)(-10+10)=0$

$\frac{x^2+3x+13}{x^2-7x-18}$

Is 2 an excluded Value?

$2^2-7(2)-18 = 4-14-18 = -28$ NO

what about -2?

$(-2)^2-7(-2)-18 = 4+14-18=0$ Yes

How to find excluded Value:

1) Denominator = 0

2) Solve

Find all excluded values:

$\frac{2x-7}{3x+5}$

$3x+5=0$

$3x=-5$

$x=-5/3$

E.V.: $\frac{-5}{3}$

$\frac{x+8}{(x-4)(x+7)}$

$(x-4)(x+7)=0$

Zero-Product Rule

$x-4=0$ $\quad x+7=0$

$x=4$ $\quad x=-7$

E.V. $4, -7$

$\frac{2x-9}{x^2-16}$, Find E.V.: $x^2-16=0$

$(x+4)(x-4)=0$

Use Z.F.P.

$x+4=0$ $\quad x-4=0$

$\boxed{x=-4}$ $\quad \boxed{x=4}$

E.V.: ± 4

$\frac{?}{2x^2-5x-7}$ Find all excluded Values

$2x^2-5x-7=0$

$(2x-7)(x+1)=0$

Z.F.P. $\quad 2x-7=0$ $\quad x+1=0$

$x=\frac{7}{2}$ $\quad x=-1$

E.V. $\frac{7}{2}, -1$

Find all excluded values:

$$\frac{3x+8}{2x^2+x-3}$$

→ Denom. = 0, Solve

$2x^2 + x - 3 = 0$

$a=2,\ b=1,\ c=-3$

$b^2-4ac = (1)^2-4(2)(-3) = 25$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{25}}{4}$$

$$= \frac{-1 \pm 5}{4}$$

$$x = \frac{-1+5}{4} = 1$$

$$x = \frac{-1-5}{4} = -\frac{3}{2}$$

E.V.

$1 \text{ & } -\frac{3}{2}$

Find all excluded values:

$$\frac{7x-2}{3x^2-2x-5}$$

→ Denom. = 0, Solve

$3x^2-2x-5=0$

Factoring

$(3x-5)(x+1)=0$

$x=\frac{5}{3}$ $\quad x=-1$

E.V.: $\frac{5}{3}$ & -1

Q-formula

$a=3,\ b=-2,\ c=-5$

$b^2-4ac=(-2)^2-4(3)(-5)=64$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{2 \pm \sqrt{64}}{6}$$

$$= \frac{2 \pm 8}{6}$$

$$x = \frac{2+8}{6}$$

$\boxed{x = \frac{5}{3}}$

$$x = \frac{2-8}{6}$$

$\boxed{x=-1}$

Find all excluded values:

$$\frac{x^2-25}{3x^2+13x+4}$$

Factoring

$(3x+1)(x+4)=0$

$x=-\frac{1}{3}$ $x=-4$

E.V. $-\frac{1}{3}$ & -4

Deno = 0, Solve

$3x^2+13x+4=0$

$a=3$ $b=13$ $c=4$

$b^2-4ac=(13)^2-4(3)(4)$

$=169-48=121$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-13\pm\sqrt{121}}{6}=\frac{-13\pm11}{6}$$

$x=\frac{-13+11}{6}=-\frac{1}{3}$ $x=\frac{-13-11}{6}=-4$

Find all excluded values

$$\frac{7x}{(3x-5)^2-81}$$

Deno. = 0, Solve

$(3x-5)^2-81=0$

$(3x-5)^2=81$

Square-Root method

$3x-5=\pm\sqrt{81}$

$3x-5=\pm9$

$3x-5=9$ $3x-5=-9$

$x=\frac{14}{3}$ $x=\frac{-4}{3}$

E.V. $\frac{14}{3}$ & $\frac{-4}{3}$

Find all excluded values:

$$\frac{2x+5}{(2x+5)^2-49}$$

→ Denom. = 0, Solve

$(2x+5)^2 - 49 = 0$

$(2x+5)^2 = 49$

use S.R.M.

$2x+5 = \pm\sqrt{49}$

$2x+5 = \pm 7$

$2x+5 = 7$ or $2x+5 = -7$

$x = 1$ $\quad$ $x = -6$

E.V.

$1, -6$

How to multiply rational expressions:

1) Factor everything Completely

2) Cross-out any common factors vertically or diagonally.

3) Write the remaining factors in product form

$$\frac{x^2-25}{x^2-8x+15} \cdot \frac{x^2-9}{x^2+3x} = \frac{(x+5)(x-5)}{(x-3)(x-5)} \cdot \frac{(x+3)(x-3)}{x(x+3)}$$

$$= \frac{x+5}{x}$$

Multiply:

$$\frac{8x^2-18}{2x^2-5x+3}\cdot\frac{x^2-9x+8}{6x^2+7x-3}=\frac{2(4x^2-9)}{(2x-3)(x-1)}\cdot\frac{(x-8)(x-1)}{(3x-1)(2x+3)}$$

(A^2-B^2)

$6x^2+7x-3$: $S=7$, $P=-18$, -18, $-2,9$

$6x^2-2x+9x-3$

$2x(3x-1)+3(3x-1)$

$$=\frac{2(2x+3)(2x-3)}{(2x-3)(x-1)}\cdot\frac{(x-8)(x-1)}{(3x-1)(2x+3)}$$

$$=\boxed{\frac{2(x-8)}{3x-1}}$$

Simplify:

$$\frac{7x^2-35}{x^2-25}\cdot\frac{x^2+3x-10}{x^2+4x}\cdot\frac{7x^2+14x-56}{x^2-4x+4}$$

$$=\frac{7(x^2-5)}{(x+5)(x-5)}\cdot\frac{(x+5)(x-2)}{x(x+4)}\cdot\frac{7(x^2+2x-8)}{(x-2)(x-2)}$$

$$=\frac{7(x^2-5)}{x-5}\cdot\frac{1}{x(x+4)}\cdot\frac{7(x+4)(x-2)}{x-2}=\boxed{\frac{49(x^2-5)}{x(x-5)}}$$

Simplify:

$$\frac{36x^2-64}{3x^2+10x+8} \cdot \frac{x^2-5x-14}{3x^2-13x+12} \cdot \frac{x^2-9}{4x^2-16x-84}$$

$$= \frac{4(9x^2-16)}{(3x+4)(x+2)} \cdot \frac{(x-7)(x+2)}{(3x-4)(x-3)} \cdot \frac{(x+3)(x-3)}{4(x^2-4x-21)}$$

$$= \frac{4(3x+4)(3x-4)}{(3x+4)(x+2)} \cdot \frac{(x-7)(x+2)}{(3x-4)(x-3)} \cdot \frac{(x+3)(x-3)}{4(x-7)(x+3)}$$

$$= \frac{4}{4} = \boxed{1}$$

To divide rational expressions:

1) Convert division to multiplication of reciprocals
2) Proceed with Steps on how to multiply rational expressions.

$$\frac{9x+18}{4x^2-3x} \div \frac{x^2-4}{4x^2-11x+6} = \frac{9x+18}{4x^2-3x} \cdot \frac{4x^2-11x+6}{x^2-4}$$

$$= \frac{9(x+2)}{x(4x-3)} \cdot \frac{(4x-3)(x-2)}{(x+2)(x-2)} = \boxed{\frac{9}{x}}$$

Simplify:

1) $\frac{4x+4}{2x-3} \div \frac{x^2-1}{2x^2+x-6}$

$= \frac{4(x+1)}{2x-3} \cdot \frac{(2x-3)(x+2)}{(x+1)(x-1)}$

$= \boxed{\frac{4(x+2)}{x-1}}$

2) $\frac{15x+5}{3x^2-14x-5} \div \frac{15}{3x-12}$

$= \frac{5(3x+1)}{(3x+1)(x-5)} \cdot \frac{3(x-4)}{15}$

$= \boxed{\frac{x-4}{x-5}}$

Due Tuesday: SG 17

Also work on SG 18